EQUATIONS OF MECHANICAL ENERGY OF MONODISPERSE SUSPENSIONS

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UDC 536.70+532.135

Equations of the balance of mechanical energy of monodisperse suspensions have been obtained by the method of averaging over a statistical ensemble of possible spatial configurations of solid sperical particles.

At the present time nonequilibrium thermodynamics of heterogeneous media is constructed by generalization of the methods [1, 2]. A review of the results obtained in this direction can be found in [3-6]. In [7, 8] the calculation scheme contains thermodynamics of the phase separation surface. In [6] an expression is obtained for the production of entropy, and also a complete system of equations of motion of a disperse medium in which processes of heat and mass transfer take place in the presence of chemical reactions and phase transitions.

We know that to obtain an explicit form for the production of entropy, continuum equations of energy balance are necessary. However, different models of continua lead to different results reflecting both the concrete model representations and the methods of averaging the "microscopic" equations which are valid on the level of the individual phases. Recently, Buevich and his collaborators [9, 10] have developed very effective statistical methods for the derivation of continuum balance equations for observable mean motion of a mixture. They allow us to consider from a unified position the balance equations and the rheological equations of state. Already on the basis of them it has been possible to solve a fairly large number of problems of heat and mass transfer and hydrodynamics of disperse media (see Inzh.-Fiz. Zh. of recent years).

It is of interest to use a statistical ensemble [9, 10] also for the construction of nonequilibrium thermodynamics of suspensions. Since in [9, 10] only equations of balance of the impulse and moment were considered, below we shall present the derivation of equations of balance of the kinetic and potential energies.

The theory being considered is constructed on the concept of a statistical ensemble of possible spatial configurations of solid particles with the detail distribution function

$$\Phi(t, \vec{r}^{(1)} \ldots r^{(N)}), \quad \int \Phi d\vec{r}^{(1)} \ldots d\vec{r}^{(N)} = 1. \tag{1}$$

The radius vectors of particles $\vec{r}^{(i)}$ $(i = 1 \ldots N)$ vary arbitrarily within the limits of the volume occupied by the mixture (here

$$|\vec{r}^{(i)} - \vec{r}^{(j)}| \geqslant 2a, \quad i \neq j, \quad |\vec{r}^{(i)} - \vec{r}_{\text{boundary}}| \geqslant a$$

in view of the condition of spheres of radius a not being exceeded) and form a 3N-dimensional space C_N. We note that the distribution function in the phase space Φ can in the general case also depend on vectors of dipole moments of particles, but in the present paper we shall not consider the rotation of particles. The "spreading" of the properties of discrete particles over the volume is effected by means of the generalized function θ_1 connected with the Heaviside function η:

$$\theta_1(\vec{r}\,|\,C_N) = \sum_{i}^{N} \eta(a - |\vec{r} - \vec{r}^{(i)}|). \tag{2}$$

For the solid phase we introduce the generalized function

$$\theta_0(\vec{r}\,|\,C_N) = 1 - \theta_1, \quad \theta_0^2 = 1, \quad \theta_1^2 = 1. \tag{3}$$

For a local physical quantity (velocity, impulse, energy, etc.) $G(t, \vec{r}\,|\,C_N)$, defined within particles and in the liquid but in the general case discontinuous on the surface of particles, we introduce the ensemble averages

$$g(t, \vec{r}) = \langle G \rangle = \int G(t, \vec{r}\,|\,C_N)\, \Phi(t\,|\,C_N)\, dC_N, \tag{4}$$

S. M. Kirov Ural Polytechnic Institute, Sverdlovsk. Translated from Inzhenerno-Fizicheskii Zhurnal, Vol. 37, No. 4, pp. 641-648, October, 1979. Original article submitted October 23, 1978.

0022-0841/79/3704-1173$07.50

$$dC_N = d\vec{r}^{(1)} \dots d\vec{r}_{(N)},$$

$$\varepsilon(t, \vec{r})\, g_0(t, \vec{r}) = \langle \theta_0 G \rangle, \tag{4a}$$

$$\rho(t, \vec{r})\, g_1(t, \vec{r}) = (1-\varepsilon)\, g_1 = \langle \theta_1 G \rangle. \tag{4b}$$

The averaging operator commutates the operations of differentiation with respect to the hydrodynamic variables (coordinates) $\vec{r}$ and the time t. After application of the averaging operator <...> the balance equations, in addition to averages quantities, contain terms due to random pulsations of the phases and terms describing their interaction. The latter are expressed in terms of functionals of mean particles on the surface. Thus, the "macroscopic" (averaged) behavior of the mixture can be connected with the behavior at the level of individual particles, and in a certain approximation we can go over to the problem of a "test particle." In the present work we consider only mixtures of solid phase with hard particles without diffusion, chemical reactions, and phase transitions. The materials of the phases are considered incompressible. Taking into account the asymmetry of the balance equations for different phases and the complexity of the results, it is convenient to introduce the matrices

$$\boldsymbol{\Theta} = \begin{pmatrix} \theta_0 & 0 \\ 0 & \theta_1 \end{pmatrix}, \quad \mathbf{I} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}, \quad \mathbf{a} = \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}, \quad \mathbf{b} = \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix}, \tag{5}$$

with $\boldsymbol{\Theta}\mathbf{I} = \boldsymbol{\Theta}$, $\boldsymbol{\Theta}^2 = \boldsymbol{\Theta}$, $\mathrm{Sp}\,\boldsymbol{\Theta} = 1$, $\mathbf{a} + \mathbf{b} = \mathbf{I}$,

$$\mathbf{I} - \boldsymbol{\Theta} = \begin{pmatrix} \theta_1 & 0 \\ 0 & \theta_0 \end{pmatrix}, \quad \mathbf{a}\boldsymbol{\Theta} = \mathbf{a}\theta_0 = \begin{pmatrix} \theta_0 & 0 \\ 0 & 0 \end{pmatrix}, \quad \mathbf{b}\boldsymbol{\Theta} = \mathbf{b}\theta_1 = \begin{pmatrix} 0 & 0 \\ 0 & \theta_1 \end{pmatrix}. \tag{6}$$

Using the generalized density function

$$D = \theta_0 d_0 + \theta_1 d_1, \tag{7}$$

we find

$$D\boldsymbol{\Theta} = \mathbf{d}\boldsymbol{\Theta} = \begin{pmatrix} \theta_0 d_0 & 0 \\ 0 & \theta_1 d_1 \end{pmatrix}, \quad D\boldsymbol{\Theta}G = \mathbf{d}\boldsymbol{\Theta}G = \begin{pmatrix} d_0 \theta_0 G & 0 \\ 0 & d_1 \theta_1 G \end{pmatrix}. \tag{8}$$

Applying the operation of averaging (4) to the matrices Θ, (7) and (8), we obtain the phase averages

$$\langle \boldsymbol{\Theta} \rangle = \boldsymbol{\vartheta} = \begin{pmatrix} \varepsilon & 0 \\ 0 & \rho \end{pmatrix}, \quad \langle D\boldsymbol{\Theta} \rangle = \boldsymbol{\vartheta}\mathbf{d} = \mathbf{D} = \begin{pmatrix} \varepsilon d_0 & 0 \\ 0 & \rho d_1 \end{pmatrix},$$

$$\langle D\boldsymbol{\Theta}G \rangle = \mathbf{d}\boldsymbol{\vartheta}\mathbf{g} = \mathbf{D}\mathbf{g}, \tag{9}$$

$$\mathbf{d} = \begin{pmatrix} d_0 & 0 \\ 0 & d_1 \end{pmatrix}, \quad \mathbf{g} = \begin{pmatrix} g_0 & 0 \\ 0 & g_1 \end{pmatrix}.$$

The mean value G for the mixture equals

$$g = \langle G \rangle = \mathrm{Sp}\,\langle \boldsymbol{\Theta}G \rangle = \mathrm{Sp}\,\boldsymbol{\vartheta}\mathbf{g} = \varepsilon g_0 + \rho g_1. \tag{10}$$

The mean with respect to the phases can be expressed in terms of the mean for the entire mixture and the mean for the solid phase. For this we write

$$(\langle \boldsymbol{\Theta}G \rangle - \mathbf{a}\langle (\mathbf{I}-\boldsymbol{\Theta})G \rangle) + \mathbf{a}\langle (\mathbf{I}-\boldsymbol{\Theta})G \rangle.$$

Taking into account (5), (6), and (10), after algebraic transformations we obtain

$$\langle \boldsymbol{\Theta}G \rangle = \boldsymbol{\vartheta}\mathbf{g} = \mathbf{a}\,\mathrm{Sp}\,(\boldsymbol{\vartheta}\mathbf{g}) + (\mathbf{b}-\mathbf{a})\,\langle \theta_1 G \rangle. \tag{11}$$

Analogously, we can write

$$\langle \boldsymbol{\Theta}\vec{\nabla}G \rangle = \vec{\nabla}(\mathbf{a}\,\mathrm{Sp}\,(\boldsymbol{\vartheta}\mathbf{g})) + (\mathbf{b}-\mathbf{a})\,\langle \theta_1 \vec{\nabla}G \rangle. \tag{12}$$

The second term in (12) can be reduced to the integral over the surface of the test particle and it describes the contribution of the surface effects. The elements of the matrices have the tensor dimensionality 0, 1, 2. The dot (·) between matrices denotes scalar product of their elements, while : denotes the product of tensor elements contracted twice.

The matrix of generalized functions Θ is subjected to the equation

$$\frac{\partial}{\partial t}\Theta + \vec{\nabla}\cdot(\Theta\vec{V}) = 0, \tag{13}$$

with $\dot{\Theta} = \frac{d\Theta}{dt} = 0.$

When deriving (13) we have taken into account solenoidality of the field of velocities $\vec{V}$. Averaging (13) over the ensemble and taking into account (9), we obtain the equation of conservation of mass of incompressible phases

$$\frac{\partial}{\partial t}\boldsymbol{\vartheta} + \vec{\nabla}\cdot(\boldsymbol{\vartheta}\mathbf{v}) = 0, \tag{14}$$

where

$$\mathrm{Sp}\,\boldsymbol{\vartheta} = 1,$$

$$\mathbf{v} = \begin{pmatrix} \vec{v}_0 & 0 \\ 0 & \vec{v}_1 \end{pmatrix}, \quad \begin{matrix} \vec{v}_0 = \varepsilon^{-1}\langle \theta_0\vec{V}\rangle, \\ \vec{v}_1 = \rho^{-1}\langle \theta_1\vec{V}\rangle. \end{matrix}$$

We introduce the pulsation fields

$$\boldsymbol{\Theta}G = \boldsymbol{\Theta}\mathbf{g} + \boldsymbol{\Theta}G', \tag{15}$$

$$\mathbf{I}\vec{V} = \mathbf{v} + \mathbf{I}\vec{V}'.$$

Taking into account (13), the averaged value of the substantial derivative can be represented in the form (for details of the averaging procedure see [9])

$$\langle D\Theta\frac{dG}{dt}\rangle = \langle \Theta\frac{dDG}{dt}\rangle = \mathbf{d}\langle \Theta\frac{dG}{dt}\rangle =$$

$$= \frac{\partial}{\partial t}(\mathbf{dg}\boldsymbol{\vartheta}) + \vec{\nabla}\cdot(\mathbf{d}\boldsymbol{\vartheta}\mathbf{gv} + \mathbf{d}\langle \Theta G'\vec{V}'\rangle = \frac{\partial}{\partial t}(\mathbf{Dg}) + \vec{\nabla}(\mathbf{Dgv} + \mathbf{d}\langle \Theta G'\vec{V}'\rangle. \tag{16}$$

We shall average the equation of motion which is valid for each of the phases:

$$D\frac{d\vec{V}}{dt} = -\vec{\nabla}\cdot\Sigma - D\psi,$$

where Σ is the pressure tensor (in [9] the stress tensor Σ is used); ψ is the potential of the external forces. Having multiplied by Θ and carried out the averaging operation with (11), (12), and (16) ($G \equiv \vec{V}$) taken into account, we obtain

$$\frac{\partial}{\partial t}(\mathbf{Dv}) + \vec{\nabla}\cdot(\mathbf{Dvv}) = -\vec{\nabla}\cdot\boldsymbol{\sigma}_a + \mathbf{f}_a - \mathbf{D}\vec{\nabla}\psi. \tag{17}$$

Here $\boldsymbol{\sigma}_a = \mathbf{a}\,\mathrm{Sp}(\boldsymbol{\vartheta}\boldsymbol{\sigma}) + \boldsymbol{\sigma}^*$, $\mathbf{f}_a = (\mathbf{b} - \mathbf{a})\vec{f}$; $\boldsymbol{\sigma} = \begin{pmatrix} \sigma_0 & 0 \\ 0 & \sigma_1 \end{pmatrix}$, matrix of mean stresses of the phases; $\boldsymbol{\sigma}^* = \mathbf{d}\langle \Theta\vec{V}'\vec{V}'\rangle = \begin{pmatrix} d_0\langle \theta_0\vec{V}'\vec{V}'\rangle & 0 \\ 0 & d_1\langle \theta_1\vec{V}'\vec{V}'\rangle \end{pmatrix}$, matrix of the actual pulsating stresses; $\vec{f} = -\langle \theta_1\vec{\nabla}\cdot\Sigma\rangle$, mean hydrodynamic force acting on particles in a unit volume; $\vec{V}'\vec{V}'$, diadic product of the pulsating constituents of the velocity field. The matrix equation (17) formally coincides with the equation of balance of impulse for a single-phase medium, and is completely equivalent to the system of equations obtained by Buevich and Shchelchkova in [9, 10]. If we take into account the pulsations of the external field, then (17) will contain additional terms, while instead of ψ there enters the matrix Ψ which will be defined below.

We multiply all terms of Eq. (17) by the matrix **v**. After standard transformations we obtain the equation of balance of the translational kinetic energy of the averaged motion of the disperse medium

$$\frac{\partial}{\partial t}\left(\frac{1}{2}\mathbf{Dv}^2\right) + \vec{\nabla}\cdot\left(\frac{1}{2}\mathbf{Dv}^2\mathbf{v}\right) = -\vec{\nabla}\cdot(\boldsymbol{\sigma}_a\cdot\mathbf{v}) + \boldsymbol{\sigma}_a\!:\!\vec{\nabla}\mathbf{v} + \mathbf{f}_a\mathbf{v} - \mathbf{Dv}\cdot\vec{\nabla}\psi. \tag{18}$$

Here $\boldsymbol{\sigma}_a\cdot\mathbf{v} = \mathbf{a}\,\mathrm{Sp}(\boldsymbol{\vartheta}\boldsymbol{\sigma})\cdot\mathbf{v} + \boldsymbol{\sigma}^*\cdot\mathbf{v}$; $\boldsymbol{\sigma}_a : \vec{\nabla}\mathbf{v} = \mathbf{a}\,\mathrm{Sp}(\boldsymbol{\vartheta}\boldsymbol{\sigma}) : \vec{\nabla}\mathbf{v} + \boldsymbol{\sigma}^* : \vec{\nabla}\mathbf{v}$.

Equation (18) also formally coincides with the equation of balance of the kinetic energy of the averaged motion of a single-phase medium, and its terms have analogous physical meaning.

To isolate the contribution of the pulsating constituents to the balance of kinetic energy, we carry out averaging of the equation of balance of the translational kinetic energy which is valid at the level of the individual phases:

$$D\frac{d}{dt}\left(\frac{1}{2}V^2\right)=-\vec{\nabla}\cdot\Sigma\cdot\vec{V}+\Sigma:\vec{\nabla}\vec{V}-D\vec{V}\cdot\vec{\nabla}\psi. \tag{19}$$

We multiply (19) term by term by Θ and, putting in (15) $G \equiv \Sigma$, ψ, we introduce it into the pulsation equation. The application of the averaging (4) yields the following. On the left side (19), with (16) taken into account

$$\begin{gathered}\frac{1}{2}\langle 2D\Theta\frac{d}{dt}\vec{V}\cdot\vec{V}\rangle=\frac{1}{2}\langle D\Theta\frac{d}{dt}(\mathbf{v}^2+2\mathbf{v}\cdot\vec{V}'+\mathbf{I}\vec{V}'\cdot\vec{V}')\rangle=\frac{\partial}{\partial t}\left[\frac{1}{2}\mathbf{D}\mathbf{v}^2+\frac{1}{2}\mathbf{d}\langle\mathbf{\Theta}(\vec{V}'\cdot\vec{V}')\rangle\right]+\\+\vec{\nabla}\cdot\left[\frac{1}{2}\mathbf{D}\mathbf{v}^2\mathbf{v}+\mathbf{d}\langle\mathbf{\Theta}\vec{V}'\vec{V}'\rangle\cdot\mathbf{v}+\frac{1}{2}\mathbf{v}\mathbf{d}\langle\mathbf{\Theta}(\vec{V}\cdot\vec{V}')\rangle+\frac{1}{2}\mathbf{d}\langle\mathbf{\Theta}\vec{V}'(\vec{V}'\cdot\vec{V}')\rangle\right].\end{gathered} \tag{20}$$

On the right side, by means of (11) $(G=(\mathbf{\Theta}\boldsymbol{\sigma}+\mathbf{\Theta}\Sigma')(\mathbf{v}+\mathbf{I}\vec{V}'))$, (12) and (10), the sum of the first two terms is represented as:

$$\begin{gathered}-\langle(\mathbf{\Theta}\vec{\nabla}\cdot\Sigma\cdot\vec{V}\rangle+\langle\mathbf{\Theta}\Sigma:\vec{\nabla}\vec{V}\rangle=-\mathbf{a}\vec{\nabla}\cdot\langle\Sigma\cdot\vec{V}\rangle+\mathbf{a}\langle\Sigma:\vec{\nabla}\vec{V}\rangle-\\-(\mathbf{b}-\mathbf{a})\langle\mathbf{\Theta}_1(\vec{\nabla}\cdot\Sigma)\cdot\vec{V}\rangle=-\mathbf{a}\vec{\nabla}[\mathrm{Sp}(\boldsymbol{\vartheta}\boldsymbol{\sigma}\cdot\mathbf{v})+\mathbf{a}\langle\Sigma'\cdot\vec{V}'\rangle+\\+\mathbf{a}\,\mathrm{Sp}(\boldsymbol{\vartheta}\boldsymbol{\sigma}:\vec{\nabla}\mathbf{v})+\mathbf{a}\langle\Sigma':\vec{\nabla}\vec{V}'\rangle+(\mathbf{b}-\mathbf{a})\vec{f}\cdot\mathbf{v}+(\mathbf{a}-\mathbf{b})\langle\theta_1(\vec{\nabla}\cdot\Sigma)\cdot\vec{V}'\rangle.\end{gathered} \tag{21}$$

The last term in (19), with pulsations of the external force field taken into account, equals

$$\langle D\mathbf{\Theta}\vec{V}\cdot\vec{\nabla}\psi\rangle=\mathbf{D}\mathbf{v}\cdot\vec{\nabla}\mathbf{\Psi}+\mathbf{d}\langle\mathbf{\Theta}\vec{V}'(\vec{\nabla}\cdot\boldsymbol{\psi}')\rangle+\mathbf{d}\langle\mathbf{\Theta}\vec{\nabla}\boldsymbol{\psi}'\rangle\cdot\mathbf{v},\quad \mathbf{\Psi}=\begin{pmatrix}\psi_0 & 0\\ 0 & \psi_1\end{pmatrix}. \tag{22}$$

Summing (21), (22) and equating the result to (20), after simple transformations we obtain the equation of balance of the translational kinetic energy of the phases with pulsations taken into account

$$\begin{gathered}\frac{\partial}{\partial t}\left(\frac{1}{2}\mathbf{D}\mathbf{v}^2+\frac{1}{2}\mathbf{d}\langle\mathbf{\Theta}(\vec{V}'\cdot\vec{V}')\rangle\right)+\vec{\nabla}\cdot\left[\frac{1}{2}\mathbf{D}\mathbf{v}^2\mathbf{v}+\right.\\\left.+\frac{1}{2}\mathbf{d}\mathbf{v}\langle\mathbf{\Theta}(\vec{V}'\cdot\vec{V}')\rangle+\frac{1}{2}\mathbf{d}\langle\mathbf{\Theta}\vec{V}'(\vec{V}'\cdot\vec{V}')\rangle\right]=-\vec{\nabla}\cdot\mathbf{a}\,\mathrm{Sp}(\boldsymbol{\vartheta}\boldsymbol{\sigma}\cdot\mathbf{v})+\\+\boldsymbol{\sigma}^*\cdot\mathbf{v}+\mathbf{a}\langle\Sigma'\cdot\vec{V}'\rangle]+(\mathbf{b}-\mathbf{a})\vec{f}\cdot\mathbf{v}+(\mathbf{a}-\mathbf{b})\langle\theta_1(\vec{\nabla}\cdot\Sigma)\cdot\vec{V}'\rangle+\mathbf{a}\,\mathrm{Sp}(\boldsymbol{\vartheta}\boldsymbol{\sigma}:\vec{\nabla}\mathbf{v})+\\+\mathbf{a}\langle\Sigma':\vec{\nabla}\vec{V}'\rangle-\mathbf{D}\mathbf{v}\cdot\vec{\nabla}\mathbf{\Psi}-\mathbf{d}\langle\mathbf{\Theta}\vec{V}'\cdot\vec{\nabla}\boldsymbol{\psi}'\rangle-d\langle\mathbf{\Theta}\vec{\nabla}\boldsymbol{\psi}'\rangle\cdot\mathbf{v}.\end{gathered} \tag{23}$$

In (23) we have taken into account the fact that $\boldsymbol{\sigma}^*=\mathbf{d}\langle\mathbf{\Theta}\vec{V}'\vec{V}'\rangle$ (17). Formally in the case $\mathbf{\Theta}\to\mathbf{a}$, (23) is transformed into the equation of balance of mechanical energy of a single-phase medium [11]. The physical meaning of the terms on the left side of (23) is: behind the sign $\partial/\partial t$ there is a sum of the kinetic energy of average and pulsating motions; behind the sign $\vec{\nabla}$: the first term is the density of flow of the translational kinetic energy of the averaged motion, the second term is the density of flow of the pulsating energy with averaged motion of the phases, the third term is the density of flow of the kinetic energy with pulsating motion of the phases. The terms on the right side are: behind the sign $\vec{\nabla}$ there are the densities of flow of energy due to the work of mean molecular stresses and their pulsations; the densities of flow of energy due to the work of effective pulsation stresses; the term with $\vec{f}$ equal to the power of the force of hydrodynamic interaction of the phases; the terms including products of stress tensors and velocity gradients, twice contracted, describe the powers due to the forces of pressure and viscous friction; and, finally, the last terms constitute the powers of the external mass forces and their pulsations.

We shall take into account in (18) the pulsations of the external force field, having replaced $\mathbf{D}\mathbf{v}\cdot\vec{\nabla}\psi$ with $-\mathbf{D}\mathbf{v}\cdot\vec{\nabla}\mathbf{\Psi}-\mathbf{d}\langle\mathbf{\Theta}\vec{\nabla}\boldsymbol{\psi}'\rangle\cdot\mathbf{v}$; then, subtracting term by term (18) from (23), we obtain the equation of balance of the pulsation energy of the phases

$$\begin{gathered}\frac{\partial}{\partial t}\left(\frac{1}{2}\mathbf{d}\langle\mathbf{\Theta}\vec{V}'\cdot\vec{V}'\rangle\right)+\vec{\nabla}\cdot\left[\frac{1}{2}\mathbf{d}\mathbf{v}\langle\mathbf{\Theta}\vec{V}'\cdot\vec{V}'\rangle+\frac{1}{2}\mathbf{d}\langle\mathbf{\Theta}\vec{V}'(\vec{V}'\cdot\vec{V}')\rangle\right]=-\mathbf{a}\vec{\nabla}[\mathrm{Sp}(\boldsymbol{\vartheta}\boldsymbol{\sigma}\cdot\mathbf{v})+\mathrm{Sp}(\boldsymbol{\vartheta}\boldsymbol{\sigma}\cdot\mathbf{v})]+\\+(\mathbf{a}-\mathbf{b})\langle\Theta_1(\vec{\nabla}\cdot\Sigma)\cdot\vec{V}'\rangle+\mathbf{a}\langle\Sigma':\vec{\nabla}\vec{V}'\rangle-\mathbf{d}\langle\mathbf{\Theta}\vec{V}'\cdot\vec{\nabla}\boldsymbol{\psi}'\rangle-\boldsymbol{\sigma}^*:\vec{\nabla}\mathbf{v}.\end{gathered} \tag{24}$$

The last term in (24), just as in the case of a single-phase medium, describes the mutual transformation of energy of the pulsational and averaged motions.

We carry out averaging of the equation of balance of the potential energy

$$D\frac{d\psi}{dt} = D\vec{V}\cdot\vec{\nabla}\psi. \tag{25}$$

After multiplication by θ and averaging we obtain

$$\frac{\partial}{\partial t}(\mathbf{D}\boldsymbol{\Psi}) + \vec{\nabla}\cdot(\mathbf{D}\mathbf{v}\boldsymbol{\Psi} + \mathbf{d}\langle \boldsymbol{\Theta}\psi'\vec{V}'\rangle) = \mathbf{D}\mathbf{v}\cdot\vec{\nabla}\boldsymbol{\Psi} + \mathbf{d}\langle \boldsymbol{\Theta}\vec{\nabla}\psi'\rangle\cdot\mathbf{v} + \mathbf{d}\langle \boldsymbol{\Theta}\vec{V}'\cdot\vec{\nabla}\psi'\rangle. \tag{26}$$

In order to go over to equations of balance for the mixture as a whole, we have to compute Sp of all terms of the matrix equations (17), (18), (23), (24), and (26). In view of the cumbersomeness of the results, we shall write out separately the equations of balance only for the kinetic energy (23). Taking into account the explicit form of the matrices entering into (23), we obtain for the solid phase

$$\begin{aligned}
&\frac{\partial}{\partial t}\left(\frac{1}{2}\varepsilon d_0\vec{v}_0^2 + \frac{1}{2}d_0\langle \theta_0\vec{V}'\cdot\vec{V}'\rangle\right) + \vec{\nabla}\cdot\Big[\frac{1}{2}\varepsilon d_0\vec{v}_0^2\vec{v} + \\
&+\frac{1}{2}\vec{v}_0 d_0\langle \theta_0\vec{V}'\cdot\vec{V}'\rangle + \frac{1}{2}d_0\langle \Theta_0\vec{V}'(\vec{V}'\cdot\vec{V}')\rangle\Big] = -\vec{\nabla}\cdot(\varepsilon\sigma_0\cdot\vec{v}_0 + \\
&+\rho\sigma_1\cdot\vec{v}_1 + \sigma_0^*\cdot\vec{v}_0 + \langle \Sigma'\cdot\vec{V}'\rangle) - \vec{f}\cdot\vec{v}_0 + \langle \theta_1(\vec{\nabla}\cdot\Sigma)\vec{V}'\rangle + \\
&+\varepsilon d_0\vec{v}_0\cdot\vec{\nabla}\psi_0 - \langle \theta_0\vec{\nabla}\psi'\rangle\cdot\vec{v}_0 - d_0\langle \theta_0\vec{V}'\cdot\vec{\nabla}\psi'\rangle + \varepsilon\sigma_0 : \vec{\nabla}\vec{v}_0 + \rho\sigma_1 : \vec{\nabla}\vec{v}_1 + \langle \Sigma' : \vec{\nabla}\vec{V}'\rangle,
\end{aligned} \tag{27}$$

and for particles

$$\begin{aligned}
&\frac{\partial}{\partial t}\left(\frac{1}{2}\rho d_1\vec{v}_1^2 + \frac{1}{2}d_1\langle \theta_1(\vec{V}'\cdot\vec{V}')\rangle\right) + \vec{\nabla}\cdot\Big[\frac{1}{2}\rho d_1\vec{v}_1^2\vec{v}_1 + \\
&+\frac{1}{2}\vec{v}_1 d_1\langle \theta_1(\vec{V}'\cdot\vec{V}')\rangle + \frac{1}{2}d_1\langle \theta_1\vec{V}'(\vec{V}'\cdot\vec{V}')\rangle\Big] = -\vec{\nabla}\cdot(\sigma_1^*\cdot\vec{v}_1) + \\
&+\vec{f}\cdot\vec{v}_1 - \langle \theta_1(\nabla\cdot\Sigma)\vec{V}'\rangle - \rho d_1\vec{v}_1\cdot\nabla\psi_1 + d_1\langle \theta_1\vec{\nabla}\psi'\rangle\cdot\vec{v}_1 - d_1\langle \theta_1\vec{V}'\cdot\vec{\nabla}\psi'\rangle.
\end{aligned} \tag{28}$$

In contrast to the equations of balance encountered in the literature, in the given case [9, 10] the powers of the forces of pressure and dissipation as a result of viscous friction enter into Eq. (24) for the solid phase. This is connected with the fact that the presence of particles in the flow leads to the appearance of the effective stresses $\langle\varepsilon\rangle = \sigma = \varepsilon\sigma_0 + \rho\sigma_1$, caused by the perturbations being introduced by the particles.

The next step of the computations is connected with concretization of the quantities σ and $\vec{f}$. In addition, the equations thus obtained are of interest in themselves; they allow us to go over to analysis of entropy balance in a suspension.

NOTATION

θ_0 and θ_1, generalized functions; G, a specific local quantity; g, mean value of G for the mixture; ε, porosity of the mixture; ρ, mean volume concentration of particles; $\boldsymbol{\theta}$, $\mathbf{I}$, $\mathbf{a}$, $\mathbf{b}$, matrices introduced into (5); D, generalized density function; $\boldsymbol{\vartheta}$, $\mathbf{d}$, $\mathbf{D}$, $\mathbf{g}$, matrices defined in (9); $\vec{V}$, local velocity; $\mathbf{v}$, matrix of mean velocities; Σ, local stress tensor; $\boldsymbol{\sigma}$, matrix of mean stresses; $\boldsymbol{\sigma}^*$, matrix of effective pulsation stresses; ψ, potential of an external field; $\vec{f}$, force of hydrodynamic interaction of phases; d_0, d_1, densities of materials of phases. Indices: <...>, averaging over an ensemble; 0, solid phase; 1, solid particles; $\cdot$, scalar product; :, tensor product twice contracted; a prime stands for fluctuation of a local quantity.

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SOME PROBLEMS OF HEAT AND MASS EXCHANGE OF A GAS SUSPENSION WITH RECUPERATIVE ENERGY SUPPLY

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UDC 536.245:541.182

The problem of heat exchange between a stream of a gas suspension and a third heat-transfer agent with recuperative energy supply is formulated and solved numerically.

The field of application of apparatus containing streams of a gas suspension with conductive heat supply is extensive. It is sufficient to cite heat-exchange apparatus for drying (pneumatic pipes with outside heating and cooling), recuperators, and special cases of intensification of the heat-transfer process using an intermediate disperse packing. Some types of apparatus of this kind are presented in [1]. In the present report we discuss apparatus containing coaxially arranged cylinders: a gas suspension moves through the inner channel and a heat-transfer agent (gaseous or liquid) moves through the annular outer channel (Fig. 1a). Consequently, we are talking about heat exchange between three heat-transfer agents. Various cases of heat exchange between three heat-transfer agents with a constant heat-exchange surface are discussed in [2], andwith variable heat-exchange surfaces in [3]. In the latter case disperse particles are used as two of the heat-transfer agents. A special feature of the calculation of such systems is that the surface of each of the disperse heat-transfer agents is a variable quantity (in time or along a coordinate). Since the laws of variation of the surfaces are the same, their ratio is represented in the form of a linear dependence, which simplifies the solution of the problem. When a drying process or a drying process combined with cooling is being performed, most often only one heat-transfer agent is disperse (the material being dried or cooled), and its surface is a variable quantity; the second heat-exchange surface, the surface of the cylinder, while it varies with the coordinate, does so by a different law from that for the surface of the disperse heat-transfer agent, so that their ratio is expressed by a complicated dependence. This fact introduces a new and complicating element into the solution of the problem.

As is known, the study of the kinetics of heating of the material, i.e., the determination of its temperature, which is an integral index of the process and at the same time is connected with the moisture content of the material, is very important in the drying process. Since the most pronounced variation in the temperature of the material occurs in the period of a declining drying rate, only this period is considered below.

Neglecting the gradients of temprature and moisture content over the cross section of a particle and assuming the absence of a temperature gradient over the cross section of the stream of gas suspension (i.e., developed turbulent flow of the gas suspension occurs), we

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0022-0841/79/3704-1178$07.50